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ner described above, then it is possible to complete the figure of the fifteen points  $LM, LN, \dots QR$  and of the twenty lines  $LMN, LMP, \dots PQR$ , such that through each point pass four lines, and on each line lie three points, as detailed in the foregoing.

Of the fifteen points, nine, viz. the points  $LP, LQ, LR; MP, MQ, MR; NP, NQ, NR$  are, as appeared above, points on two of the six lines 1, 2, 3, 4, 5, 6; the remaining points are  $MN, NL, LM; QR, RP, PQ$ . These are *Brianchon points*

$MN$ of the six-side	162435
$NL$	" 152634
$LM$	" 142536
$QR$	" 152436
$RP$	" 142635
$PQ$	" 162534,

for the point  $MN$  is the meet of lines  $MNP, MNQ, MNR \equiv MP, NP; MQ, NQ; MR, NR \equiv 35, 26; 16, 34; 24, 15$ ; that is,  $MN$  is the Brianchon point of the six-side 162435; and similar reasoning verifies the above statements for the rest of the six-lines.

To summarize, we have two sets of three six-sides such that the Brianchon points of each set lie *in linea*; and the two lines so obtained together with the eighteen lines through the six Brianchon points, form a system of twenty lines passing by fours through fifteen points.

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## DEPARTMENTS.

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### SOLUTIONS OF PROBLEMS.

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#### ALGEBRA.

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154. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Deduce the Sylvestrian Reciprocant of  $ax^3 + 3bx^2y^2 + ay^3 + d = 0$ .

Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Differentiating, dividing by 3, and combining, we have

$$a(x^2 + y^2 \frac{dy}{dx}) + 2bxy(y + x \frac{dy}{dx}) = 0 \dots (1).$$

Repeating the operation, we have

$$a \left[ x + 2y \left( \frac{dy}{dx} \right)^2 + y^2 \frac{d^2 y}{dx^2} \right] + 2b \left[ y^2 + 4xy \frac{dy}{dx} + x^2 \left( \frac{dy}{dx} \right)^2 + x^2 y \frac{d^2 y}{dx^2} \right] = 0 \dots (2).$$

Eliminating  $a$  and  $b$  in equations (1) and (2), we have

$$\begin{vmatrix} x^2 + y^2 \frac{dy}{dx}, & xy \left( y + x \frac{dy}{dx} \right) \\ x + 2y \left( \frac{dy}{dx} \right)^2 + y^2 \frac{d^2 y}{dx^2}, & \left( y + 2x \frac{dy}{dx} \right)^2 + x^2 y \frac{d^2 y}{dx^2} - \left( \frac{dy}{dx} \right)^2 \end{vmatrix} = 0,$$

which is the Sylvestrian Reciprocant of  $ax^3 + 3bx^2y^2 + ay^3 + d = 0$ , since this function would have the same form if  $x$  were the dependent and  $y$  the independent variable.

167. Proposed by G. B. M. ZERE, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A weight of  $m$  pounds falls and is broken into  $n$  pieces after which it is found that all weights, in pounds, from 1 to  $m$  can be weighed. Find the weight of each piece. Apply when  $m=121$ ,  $n=5$ .

I. Solution by the PROPOSER.

Let  $x_1, x_2, x_3, \dots, x_n$  be the  $n$  pieces. Then  $x_1 + x_2 + x_3 + \dots + x_{n-1} + x_n = m$ ,  
 $x_2 - x_1 = x_1 + 1$ , or  $x_2 = 2x_1 + 1$ .

$$x_3 - x_2 - x_1 = x_2 + x_1 + 1, \text{ or } x_3 = 2x_2 + 2x_1 + 1 = 3(2x_1 + 1) = 3x_2.$$

$$x_4 - x_3 - x_2 - x_1 = x_3 + x_2 + x_1 + 1, \text{ or } x_4 = 2x_3 + 3x_2 = 9(2x_1 + 1) = 9x_2.$$

$$\text{Generally, } x_r = 3^{r-2}(2x_1 + 1) = 3^{r-2}x_2.$$

$$\therefore x_1 + x_2 + x_3 + \dots + x_n = x_1 + (2x_1 + 1)(1 + 3 + 9 + 27 + \dots + 3^{n-2}) = m.$$

$$\therefore x_1 + (2x_1 + 1)(3^{n-1} - 1) = 2m \text{ or } x_1 \frac{2m + 1 - 3^{n-1}}{2 \cdot 3^{n-1}}.$$

$$x_2 = (2x_1 + 1) = \frac{2m + 1}{3^{n-1}}; \quad x_r = 3^{r-2}x_2 = \frac{2m + 1}{3^{n-r+1}}.$$

When  $m=121$ , and  $n=5$ ,  $x_1=1$ ,  $x_2=3$ ,  $x_3=9$ ,  $x_4=27$ ,  $x_5=81$ .

II. Solution by FRANK L. GRIFFIN, Graduate Student, The University of Chicago.

Let  $f(n)$  = number of groupings of  $n$  weights in two groups; then the maximum number giving one group a preponderance is  $f(n)/2$ .

Now,  $f(n) = 3^n - 1 \dots$  (i) [for proof see below]. Hence, to weigh all weights, in pounds, from 1 to  $m$  by using  $n$  weights, it is necessary that

$$m < \frac{3^n - 1}{2} \dots \text{(ii)}.$$

By using the  $n$  weights, 1, 3, 9, ...,  $3^{n-1}$ , all weights, in pounds, from 1 to